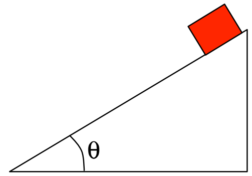
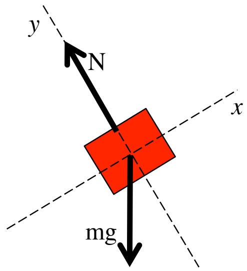
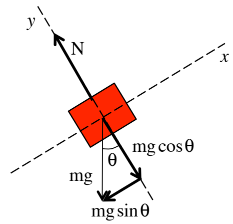
Problem 5.21

a.) The block on the fifteen degree incline accelerates down the incline. The free body diagram (f.b.d.) looks like:



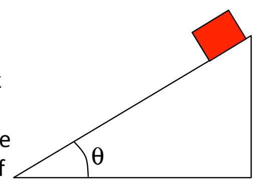


BIG NOTE: If the AP folks ask for a f.b.d., you need to draw it as it is shown above WITHOUT breaking forces into components. Later, on your own, you can redraw it to look like



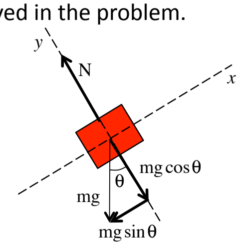
b.) What's the block's acceleration?

THIS IS EASY, which means we could just blast through it merrily, or we could get down and look to see how N.S.L. really works. Because it would be nice to just DO later, harder problem, this is the one I'm going to use to lay out the ins-and-outs of solving this class of problem.



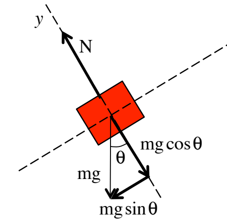
There are two ways to do these problems, the QUICK AND DIRTY way and the FORMAL way. The QUICK AND DIRTY way is how you'll want to do them on your AP test because that approach is the fastest. It doesn't, unfortunately, work easily on all problems (but does on many, many). The FORMAL way *always works* no matter what complications are involved in the problem. Knowing how to negotiate both ways is important.

In either case, you are going to have to draw a f.b.d.. It should look something like the one you see to the right.



The QUICK AND DIRTY approach.

The idea is simple. Identify the forces that are actually motivating the system to accelerate. Keeping track of signs (I talk about that at the bottom of the page), "add" up all of these forces and put them equal to the TOTAL MASS being accelerated in the system times that acceleration.



For this problem, the component "$mg\sin\theta$" is what is accelerating the system, so we write:

$$\underline{\sum F_{accelerating} = ma_x}$$
$$\Rightarrow \quad mg\sin\theta = ma$$
$$\Rightarrow \quad a_x = g\sin\theta$$
$$= \left(9.80 \text{ m/s}^2\right)\sin 15^\circ$$
$$= 2.54 \text{ m/s}^2$$

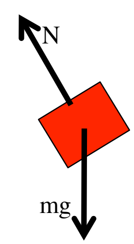
Things get a bit trickier when you have some forces pushing the system in one direction and some in another (in that case, forces in one direction are assigned to be "positive" while forces in the other "negative"), but on the whole this approach is very quick and very useful.

The FORMAL approach.

This always works and actually has steps to it. (Note that once you get comfortable with the technique, you will execute these steps without enumeration.): As laid out in Physics With Calculus (written by yours truly):

Step 0: Look at the problem and think to yourself, "I couldn't possibly figure this out." (This is here because it is a thought that will occasionally go through your head, and you might as well acknowledge it, then start the process.)

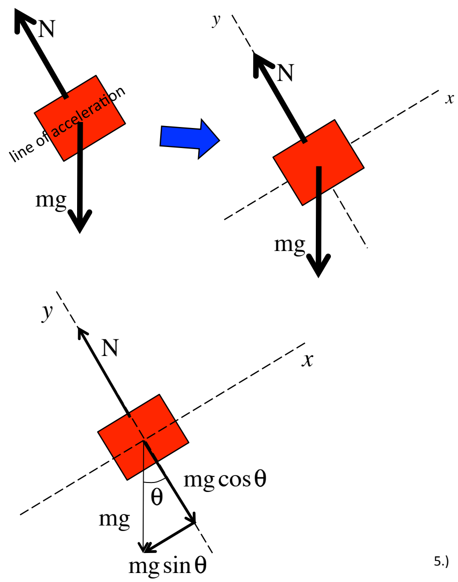
Step 1: Being sure you orient the body as it is presented in the system (that is, if the object is tilted, tilt it in the f.b.d.), pick one body in the system and draw a free body diagram for it.



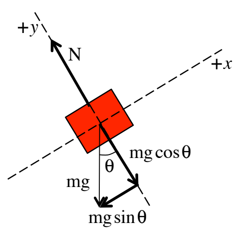
Step 2: If the first question was, "Draw a f.b.d. for the system," DO NOT DO ANY OF THE FOLLOWING STEPS ON THAT F.B.D. Instead, draw a new one--one that you can mess with. On *that* f.b.d., identify the *line of acceleration* and put one coordinate axis along that line. Put a second axis along the line perpendicular to it.

NOTE: This step is going to be REALLY important when we start dealing with centripetal situations.

Step 3: If any of the forces are off-axis, break them into their component parts along axis.



Step 4: Take one axis and sum the forces along that line. Identify the forces with algebraic variables that depict their *magnitude*; denote their direction using positive or negative signs, depending up whether the vector is in the defined positive direction or the defined negative direction. Put that sum equal to "ma," where "a" is the magnitude of the acceleration along that axis. Doing all that in this case yields:



$$\underline{\sum F_x:}$$

$$-mg\sin\theta = -ma_x$$

$$\Rightarrow \quad a_x = g\sin\theta$$

$$= 2.54\ \text{m/s}^2$$

Note 1: Notice that "a" is positive, as expected of a magnitude. The acceleration *vector*, though, was in the direction of the force, which was in the –x direction. For consistency, and to keep "a" a magnitude, the acceleration's sign must be unembedded and presented as shown. This seems like a picky point, but if you get comfortable with the idea it will get you out of a lot of trouble down the line.

Note 2: You will never write a solvable equation that has a summation sign in it, so I would suggest you use the summation sign as a blurb, as shown.

Step 5: If you have enough to solve for the variable you are looking for, you are done. If not, repeat the process for the other direction. If that doesn't provide enough information to complete the problem, you may have to repeat the entire process on another body in the system. Just keep plugging until you have what you need.

SO NOW YOU CAN SEE WHY THE *QUICK AND DIRTY* APPROACH IS PREFERABLE TO THE *FORMAL* APPROACH. THE ONE TAKES FOREVER AND HAS ALL SORTS OF PICKY LITTLE THINGS YOU HAVE TO KEEP TRACK OF. THE OTHER IS, WELL, QUICK AND DIRTY. As I said in the preamble, there will be problems in which you need to follow the Formal approach. Get to know it. You will need it on occasion.

c.) What is the speed after moving 2.00 meters down the incline?

This is a review question. Constant acceleration coupled with acceleration, change of position and initial speed allows us to use the kinematic expression:

$$(v_{x,2})^2 = (\cancel{v_{x,1}})^2 + 2a_x\Delta x \quad (v_{x,1} = 0)$$

$$\Rightarrow \quad v_2 = \sqrt{2a_x\Delta x}$$

$$= \sqrt{2(-2.54\ \text{m/s})(-2.00\ \text{m})}$$

$$= 3.18\ \text{m/s}$$